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Effective Approximation of the Photovoltaic Characteristic Curves using a Double-shaped Superellipse

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***Abstract*—Photovoltaic model (PVM) presents an easy and reliable way to evaluate both the static and dynamic behaviors of photovoltaic systems. Due to the implicitness of the basic I-V characteristic equations, a high number of unknown parameters, and the mathematical complexities of these conventional models, the effective performance analysis of these systems is still cumbersome. In this paper, a novel empirical model based on unique similarities between the geometric shapes of a superellipse and the graphical characteristics of the I-V curve is presented. As such, a step-by-step procedure describing the full-range approximation of the PV characteristic curves and parameter extraction process is illustrated. Regardless of the PV panel cell type, simulation results show that the proposed model maintains the 1% absolute error within the vicinity of the maximum power point.**

***Index Terms*—Photovoltaic, Solar cell model, I-V curve.**

I. Introduction

In recent times, solar energy has emerged as one of the cleanest and most reliable means of meeting ever-increasing energy demands [1-4]. However, harnessing this abundant energy by using photovoltaic (PV) systems can be quite difficult due to varying ambient conditions, location, and altitude of the PV panels [5-6]. To achieve the real-time modeling, simulation, and performance analysis of these systems, equivalent photovoltaic models (PVM) have been proposed in the literature.

By taking advantage of the conversion behavior describing a typical PV panel, equivalent circuit-based models and their corresponding derivatives have been successfully implemented in most power electronics software environments including MATLAB/Simulink, PSIM, PLCS, etc [7]. These models can be easily classified based on their number of diode components and subsequent fitting parameters [8].

Owing to its simplicity and fewer unknown parameters, most researchers and technicians in the industry effectively and efficiently utilize the single-diode model. However, the implicitness and nonlinearity of its basic equation make the accurate and rapid reconstruction of PV characteristic curves very tedious.

To address this challenge, numerous approximate PVM equations have been proposed in literature as reliable alternatives [7]. These PVM equations transform the basic I-V characteristic equation into simple explicit model equations by either decoupling or parameterizing its exponential term. Based on their mathematical formulas, approximate PVM equations can be classified as either analytical-based or iteration-based methods [9].

Since the solutions of the iteration-based PVM equations are heavily dependent on their initial guess values and the specified tolerance, the accuracy of the reconstructed curves within the vicinity of MPP is usually low [10-12]. As such, approximate PVM equations remain the best option in achieving the exact or near-exact replicas of the PV characteristic curves as specified in the manufacturer's datasheet [9].

The Lambert-Ω function is one of the most widely utilized methods for the decoupling of the exponential term in the basic I-V characteristic equation [13]. However, due to the mathematical complexity of the basic Lambert (Haley's) method [14-15], achieving high model accuracy, especially within the vicinity of MPP can be quite cumbersome. Several improvements in the form of series expansion formulas have been proposed for the simplification of the PVM equation [14-24].

Other analytical-based PVM equations according to literature include Taylor's series expansion [25] Padé approximant [26,27], Symbolic function [28], Chebyshev polynomials [29], Two-port network expansion [30], and two-parameter model [31]. Regardless, of the mathematical complexity of these PVM equations, the required number of unknown parameters (which are usually not readily available in the manufacturer's datasheet) are all still a hindrance to the full understanding of the behavior of PV panels. Thus, this paper proposes a novel empirical model based on the unique similarities between the graphical characteristics of the I-V curve and the geometric shape of a double-shaped superellipse.

The structure of the paper is as follows. In Section II, the conventional single-diode model and its resulting characteristic equation are extensively discussed. Afterward, the theoretical background and generalized explicit simultaneous equations describing the new proposed empirical model for PV panels are established. In Section III, the model accuracy and parameter convergence of the superellipse model is evaluated by the IEC EN 50530 standard. Finally, the conclusion and future works drawn from these results are given in Section IV.

II. Conventional Single-diode Model for Photovoltaic Panels

As shown in Fig. 1, the single-diode model consists of one diode component and five fitting parameters. By applying circuit analysis, the characteristic equation describing the typical I-V curve as shown in Fig. 2 can therefore be expressed as

$$i_{pv} = I_{ph} - I_s\left[e^{\frac{v_{pv}+i_{pv}R_s}{R_{sh}}} - 1\right] - \frac{v_{pv} + i_{pv}R_s}{R_{sh}} \qquad (1)$$

where i_{pv} is the PV output current (A), v_{pv} is the PV output voltage (V), I_{ph} is the photovoltaic current (A), I_s is the saturation current of the diode (A), V_t is the thermal voltage (V), A is the ideality factor, while R_s, R_{sh} and N is the series resistance (Ω), parallel resistance (Ω), and the number of cells in a series string inside the panel respectively.

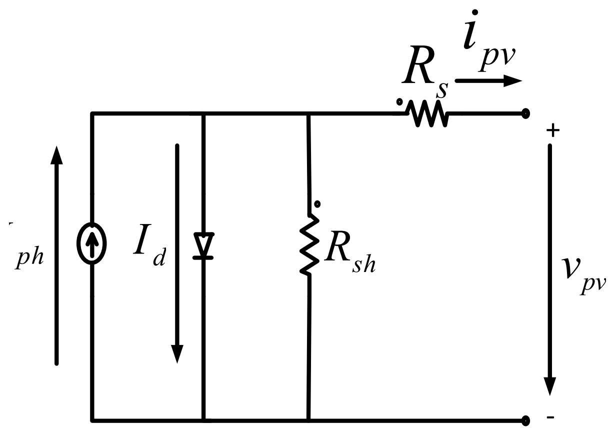

Fig. 1. Equivalent electrical circuit of the single-diode model for PV panels.

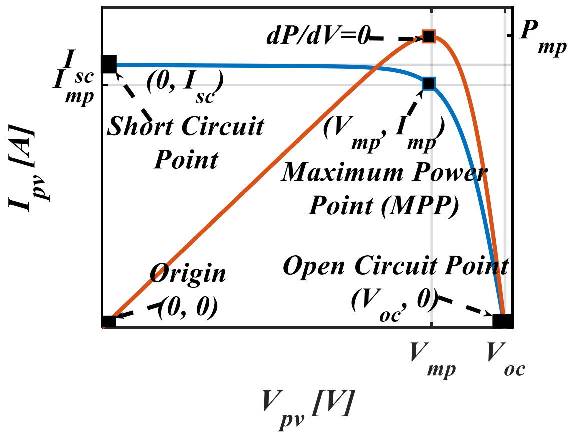

Fig. 2. A plot of the key points describing the typical PV characteristic curves.

Irrespective of ambient conditions, the typical PV characteristic curves as shown in Fig. 2 consists of four key points – voltage at the maximum power point V_{mp}, current at the maximum power point I_{mp}, open-circuit voltage V_{oc}, and short-circuit current I_{sc}.

III. Superellipse Model for Photovoltaic Panels

A. Proposed Model

Superellipse are geometrical shaped curves that constantly retains their x and y intercepts irrespective of distortion in their overall shapes as shown in Fig. 3. In its Cartesian coordinates, the implicit equation describing any point $P(x, y)$ along these curve can therefore be easily expressed as

$$\left(\frac{x}{A}\right)^m + \left(\frac{y}{B}\right)^n = 1 \qquad (2)$$

where A is the positive $x-$intercepts value, B is the positive $y-$ intercepts value, while m and n are its optimum fitting parameters.

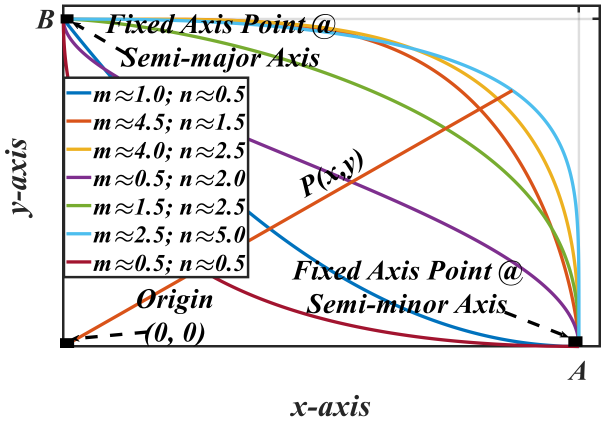

Fig. 3. A plot of a double-shaped superellipse with varying parameter values.

By taking A and B as the V_{oc} and I_{sc} of a typical I-V curve respectively, a novel implicit equation for PV panels is defined as

$$\left(\frac{v}{V_{oc}}\right)^m + \left(\frac{i}{I_{sc}}\right)^n = 1 \qquad (3)$$

where i is the output current and v is the output voltage of the superellipse model respectively.

If we make i the subject of the formula, an explicit equation describing the full-range regeneration of the I-V curve under STC can therefore be written as

$$i = I_{sc}\left[1 - \left(\frac{v}{V_{oc}}\right)^m\right]^{\frac{1}{n}}. \qquad (4)$$

B. Parameter Extraction

Due to the unique similarities between the conventional singe-diode and superellipse models, we can therefore assume both models exhibit similar mathematical properties and constraints as established by the manufacturer's PV characteristic curves. Hence, to ensure optimum parameter extraction and model accuracy within the vicinity of MPP, the following datasheet constraints are applied (4).

- Constraint 1: I-V curve enumeration starts from $(V_{oc}, 0)$ and ends at $(0, I_{sc})$. As such, the fixed points of the superellipse model; at both its semi-major and semi-minor axes are the V_{oc} and I_{sc} points respectively.
- Constraint 2: The I-V curve must always pass through its MPP. By substituting the MPP values at STC from any manufacturer's datasheet into (4), an explicit equation describing the exact MPP of the superellipse model can therefore be expressed as

$$I_{mp} = I_{sc}\left[1 - \left(\frac{V_{mp}}{V_{oc}}\right)^m\right]^{\frac{1}{n}} \qquad (5)$$

- Constraint 3: At MPP, the slope of the P-V curve is null. To meet this constraint, if we differentiate (4) such that

$$\left.\frac{dp}{dv}\right|_{v=V_{mp}} = i + v\left.\left(\frac{di}{dv}\right)\right|_{i=I_{mp},v=V_{mp}} \quad (6)$$

we obtain

$$I_{mp} = \frac{mI_{sc}}{n}\left(\frac{V_{mp}}{V_{oc}}\right)^m\left(\frac{I_{mp}}{I_{sc}}\right)^{1-n} \quad (7)$$

Therefore, by combining the mathematical equations from Constraints 2 and 3, a simultaneous equation describing the nonlinear superellipse model can be expressed as

$$I_{mp} = I_{sc}\left[1-\left(\frac{V_{mp}}{V_{oc}}\right)^m\right]^{\frac{1}{n}} \quad (8)$$

and

$$I_{mp} = \frac{mI_{sc}}{n}\left(\frac{V_{mp}}{V_{oc}}\right)^m\left(\frac{I_{mp}}{I_{sc}}\right)^{1-n} \quad (9)$$

Thus, (8) and (9) formulates the sets of necessary and sufficient conditions that must always be obeyed in extracting the fitting parameters of the superellipse model.

C. Optimization Algorithms

Several methods have been proposed in literature for obtaining the solutions to multidimensional equations [32]. Based on conventional polynomial expression, (8) and (9) can also be considered as

$$p(m,n) = I_{mp} - I_{sc}\left[1-\left(\frac{V_{mp}}{V_{oc}}\right)^m\right]^{\frac{1}{n}} \quad (10)$$

and

$$p(m,n) = I_{mp} - \frac{mI_{sc}}{n}\left(\frac{V_{mp}}{V_{oc}}\right)^m\left(\frac{I_{mp}}{I_{sc}}\right)^{1-n} \quad (11)$$

Accordingly, in this paper, two distinct approaches are considered in extracting fitting parameters of the superellipse model.

- Approach 1: By considering (10) and (11) as composite functions p and q respectively with two independent variables, the optimum fitting parameters can be obtained using the Newton-Raphson method such that

$$\begin{bmatrix} m_k \\ n_k \end{bmatrix} = \begin{bmatrix} m_{k-1} \\ n_{k-1} \end{bmatrix} - \left.\begin{bmatrix} \frac{\partial p}{\partial m} & \frac{\partial p}{\partial n} \\ \frac{\partial p}{\partial m} & \frac{\partial p}{\partial n} \end{bmatrix}\right|_{(m_{k-1},n_{k-1})}^{-1} \begin{bmatrix} p(m_{k-1},n_{k-1}) \\ q(m_{k-1},n_{k-1}) \end{bmatrix} \quad (12)$$

where $k = 1, 2, 3...$ is the iteration counts.

- Approach 2: According to algebra rules, (10) and (11) can also be equivalently rewritten as

$$\left(p(m,n)\right)^2 + \left(q(m,n)\right)^2 = 0 \quad (13)$$

Therefore, by applying optimization algorithms to (12), the superellipse fitting parameters can also be extracted. In this paper, the Powell and the Levenberg-Marquardt methods [33] are therefore utilized to minimize the objective function given by

$$f(m,n) = \left(p(m,n)\right)^2 + \left(q(m,n)\right)^2 \quad (14)$$

so that

$$f(m,n) = \left(pI_{mp} - I_{sc}\left[1-\left(\frac{V_{mp}}{V_{oc}}\right)^m\right]^{\frac{1}{n}}\right)^2 + \left(I_{mp} - \frac{mI_{sc}}{n}\left(\frac{V_{mp}}{V_{oc}}\right)^m\left(\frac{I_{mp}}{I_{sc}}\right)^{1-n}\right)^2 \quad (15)$$

IV. Parameter Convergence and Model Accuracy

A. Criteria for Evaluating Accuracy

The IEC EN50530 standard maintains the notion that the absolute current and power errors within the vicinity of $\pm 10\%$ of the PV panels V_{mp} should always be less than or equal to 1%. In this paper, the proposed empirical model is evaluated by this standard using two different PV panels – KC200GT and VBHN330SA16.

The mathematical expression used in computing these absolute errors is expressed as

$$\varepsilon_I(\%) = \frac{1}{0.2V_{mp}}\int_{V_{mp}=\pm 10\%}\left|\frac{i_e(v)-i_r(v)}{i_r(v)}\right|dv \times 100 \quad (15)$$

$$\varepsilon_P(\%) = \frac{1}{0.2V_{mp}}\int_{V_{mp}=\pm 10\%}\left|\frac{p_e(v)-p_r(v)}{p_r(v)}\right|dv \times 100$$

where the subscript e represents the expected values of the approximate curves and r denotes the data values for the reference model. Using the manufacturer's datasheet data as reference, the evaluation of the PV characteristic curves is carried out using MATLAB/Simulink in an 11th Gen Intel(R) Core(TM) i9-11900K CPU.

B. Parameter Convergence

Since equations (10) and (11) are inherently nonlinear, parameter convergence and numerical stability issues would arise if the initial values for the optimization algorithms are not well-defined. As established in Section II, the superellipse model curves have been proposed as an easy-to-fit replica of the graphical characteristics of the datasheet curves. Hence, by taking advantage of the voltage and current ratios of the typical I-V curve at STC, a mathematical expression for determining the initialization or starting values (m_0, n_0) (m_0, n_0) can therefore be expressed as

$$m_0 = \frac{V_{mp}}{V_{oc}} \quad (15)$$
$$n_0 = \frac{I_{mp}}{I_{sc}}$$

In addition, to prevent non-converging infinite iterations of the optimization algorithms in Section II. C, termination conditions are introduced such that

$$m_k - m_{k-1} \leq 1 \times 10^{-6} \qquad (16)$$
$$n_k - n_{k-1} \leq 1 \times 10^{-6}$$

with $\varepsilon_1 \approx \varepsilon_2 \approx 1 \times 10^{-6}$ in this paper.

C. Accuracy Evaluation under STC

By substituting the required PV panel specification in Table I into (10), (11) and (15) respectively, the fitting parameters of the superellipse model are easily extracted. According to the method employed in determining the search direction of the optimization algorithms, while Powell's method is considered a zero gradient technique, Newton-Raphson and Levenberg-Marquardt methods are considered second-order numerical techniques.

Although all three optimization algorithms require information (or history) from previous iteration loops, the parameter values are extracted at different iteration counts as shown in Table II. While Powell's and Newton-Raphson's methods have almost the same iteration count, the iteration count is doubled when utilizing the Levenberg-Marquardt method irrespective of PV cell type.

TABLE I
PV PANEL SPECIFICATIONS USED IN THIS PAPER

Cell Type	PV Panel	$V_{mp}\,(V)$	$I_{mp}\,(A)$	$V_{oc}\,(V)$	$I_{sc}\,(A)$
Multicrystaline	KC200GT	26.30	7.61	32.90	8.21
Ultra-thin amorphous	VBHN330SA16	58.00	5.70	69.70	6.07

TABLE II
OPTIMUM FITTING PARAMETERS FOR THE SUPERELLIPSE MODEL USING DIFFERENT ALGORITHMS.

PV Panel	Optimization Algorithm	m	n	Iteration count
KC200GT	Newton-Raphson	12.7941	0.7734	10
	Powell	12.8250	0.7690	13
	Levenberg-Marquardt	12.7840	0.7750	24
VBHN330 SA16	Newton-Raphson	15.4235	0.9630	10
	Powell	15.3770	0.9690	12
	Levenberg-Marquardt	15.4070	0.9650	26

The full-range emulations of the PV characteristic curves are therefore obtained by substituting these parameter values into (4). Figures 4 – 9 show the comparison of the superellipse model curves and the datasheet curves.

By the IEC EN 50530 standard, all reconstructed PV characteristic curves maintain the 1% absolute error within the vicinity of MPP. Regardless of the cell type, Powell's method achieves the highest model accuracy within the vicinity of MPP.

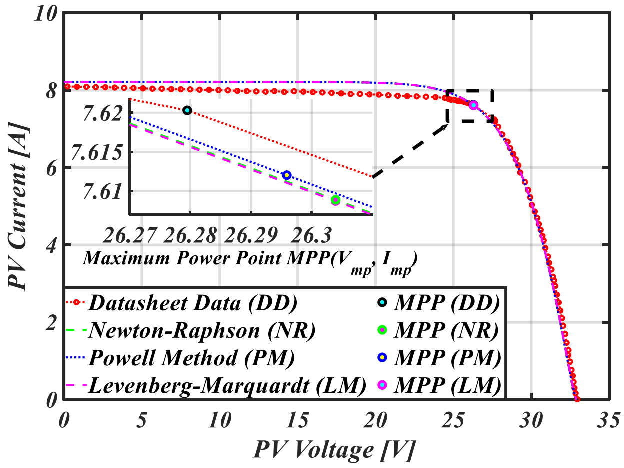

Fig. 4. Comparison of the full-range approximation of the KC200GT I-V curve.

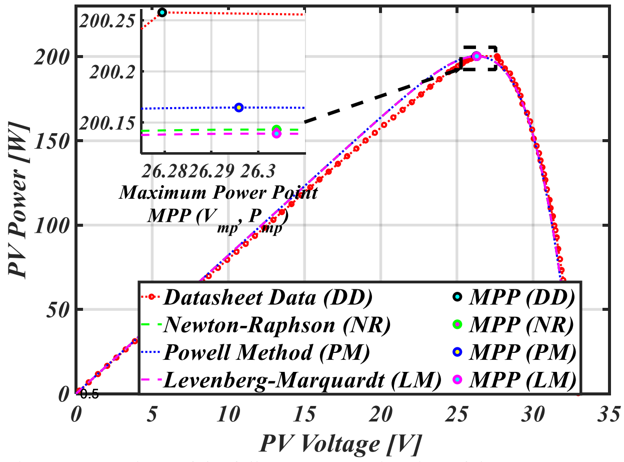

Fig. 5. Comparison of the full-range approximation of the KC200GT P-V curve.

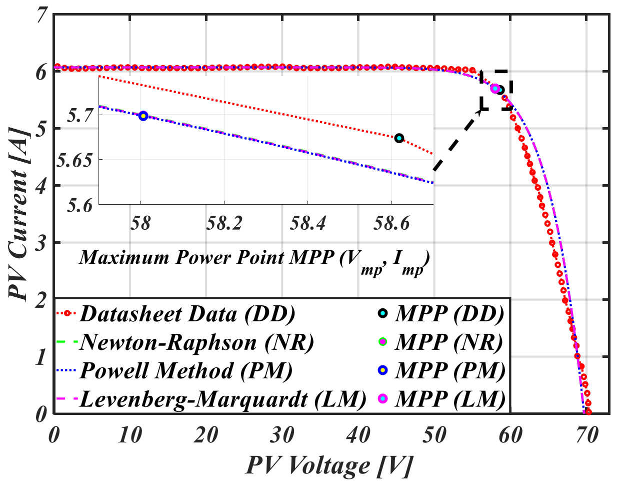

Fig. 6. Comparison of the full-range approximation of the VBHN330SA16 I-V curve.

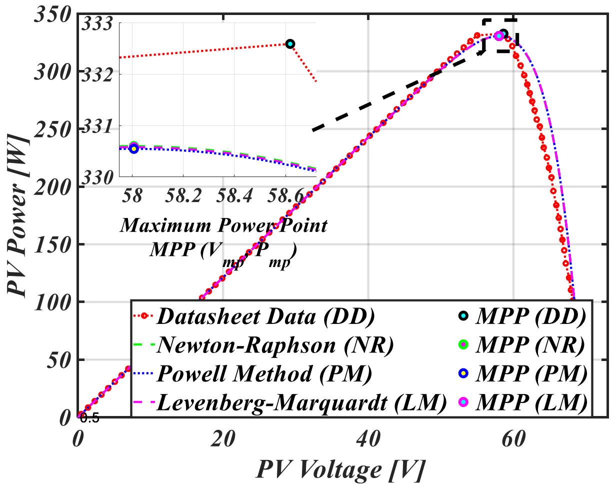

Fig. 7. Comparison of the full-range approximation of the VBHN330SA16 P-V curve.

TABLE III
ACCURACY OF THE PROPOSED MODEL WITHIN THE VICINITY OF MPP FOR TWO DIFFERENT PV PANEL

PV Panel	Optimization Algorithm	$V_{mp}\,(V)$	$I_{mp}\,(A)$	$P_{mp}\,(W)$	$\varepsilon_I\,(\%)$	$\varepsilon_P\,(\%)$
KC200GT	Datasheet Data	26.2795	7.6203	200.2577		
	Newton-Raphson	26.3039	7.6089	200.1437	0.0402	0.0153
	Powell	26.2959	7.6120	200.1647	0.0291	0.0124
	Levenberg-Marquardt	26.3134	7.6060	200.1389	0.0407	0.0158
VBHN330SA16	Datasheet Data	58.6183	5.6738	332.5882		
	Newton-Raphson	58.0067	5.6993	330.6004	0.2658	0.3529
	Powell	58.0067	5.6984	330.5463	0.2561	0.3626
	Levenberg-Marquardt	58.0067	5.6990	330.5785	0.2619	0.3568

V. CONCLUSIONS

This paper introduces a novel empirical model for evaluating the performance of PV panels. By applying the well-established datasheet constraints, explicit equations describing the full-range approximations of the PV characteristic curves and the procedures for extracting optimum fitting parameters are clearly defined. Unlike the conventional single-diode model which requires five fitting parameters at STC, the superellipse model requires only two parameters. From the simulation results and performance indices, it can be observed that irrespective of the PV panel specification or cell type, the superellipse model maintains the 1% absolute error within the vicinity of MPP.

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